Reverse Time Migration = Generalized Diffraction Stack Migration

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ABSTRACT

The standard reverse-time migration (RTM) algorithm is usually described as zero-lag correlation of the backprojected data with the source wavefield. The data are back-projected by a finite-difference algorithm, where each trace acts as a source-time history of a point source at the geophone location. This is a simple and easily understood migration method, but appears inflexible to improvement by the usual Kirchhoff tricks such as obliquity factors, first-arrival restrictions, angle-dependent truncation of data aperture or intrinsic anti-aliasing filters. In this paper, I reformulate the equations of reverse-time migration so that they can be interpreted as summing data along a series of hyperbola-like curves, each one representing a different type of event such as a reflection or multiple. This is a generalization of the familiar diffraction-stack algorithm where the migration image at a point is a sum of data along an appropriate hyperbola-like curve. For this reason I name this reformulation generalized diffraction stack migration (GDM). This formulation breathes the following new life into RTM.

1. A reverse-time common-offset migration operator is obtained without any approximation. This operator can be applied to common-offset gathers for any media, including those that induce turning waves.

2. Target-oriented RTM has been developed with the suggestion of Yi Luo.

3. A more efficient RTM method is developed that only uses first-arrival information to backpropagate arrivals. I called this wavefront wave-equation migration in a previous report.

4. GDM can be computed by any modeling method, including the reflectivity method.

5. Anti-aliasing, obliquity factors, and depth-dependent truncation of data aperture tricks can be applied to the GDM operator. In fact, a generalization of Gaussian-Beam and wavepath migration can be formulated.

6. The RTM reformulation leads to a theory for construction of the exact reverse-time migration operator from VSP data alone. No modeling computations or velocity model are needed, except simple convolutions of the VSP traces. This could be important in CDP imaging beneath salt domes where VSP data is available. In addition, migration operators for multiples can be derived from the VSP data as well.
The caveat is that the full-blown GDM can be computationally much more expensive than standard RTM, but reduced versions can make it more efficient.

**INTRODUCTION**

The two end members of migration algorithms are diffraction-stack migration (French, 1974) and reverse-time migration (McMechan, 1983). The former is computationally inexpensive, flexible, but subject to the high-frequency approximation of ray tracing. Consequently, it has difficulty with complex velocity models that preclude the accurate use of ray tracing. To overcome this limitation, reverse-time migration uses finite-difference solutions to the wave equation to accurately (Gray et al., 2001) propagate seismic energy through complex models. But the price we pay is a computationally expensive algorithm that appears inflexible to efficiency improvements or algorithmic tricks that suppress migration noise such as obliquity and intrinsic anti-aliasing filters.

To partly remediate these deficiencies of RTM, I reformulate the equations of reverse-time migration so that they can be reinterpreted as a generalized diffraction stack migration algorithm, or GDM. The simple idea behind GDM is shown in Figure 1. The top figure shows the traditional migration curve for simple diffraction stack migration. The usual interpretation is that the migration image at \( x_\theta \) is given by summing the trace amplitudes along the hyperbola, i.e., the migration image is the dot product of the focusing operator (hyperbolic curve) with the data. The bottom figure illustrates the idea behind GDM: take the dot product of the generalized focusing operator panel with the data panel. In this case, the focusing operator can be computed, for example, by a finite-difference solution to the wave equation with a point source at the image point \( x_\theta \). The traces on the surface are time shifted by the time it takes to go from the source down to the scatterer. Note that all events are included in the generalized focusing operator such as direct waves, multiples, reflections, and diffractions.

There are some benefits to GDM, including a more efficient means of carrying out RTM (Schuster, 2001; Zhou, 2002), a means for migrating common-offset gathers, and introduction of obliquity factors, intrinsic anti-aliasing filters and angle-dependent truncation of the data aperture. Finally, the GDM theory implies that we can obtain the exact focusing operator from VSP data alone. No velocity model or modeling operations are needed! Focusing operators for either primary or multiple reflections can be easily obtained by simple convolutions of traces.

This paper is divided into three parts: a theory section that reformulates reverse-time migration so that it can be interpreted as GDM, a numerical results section that applies GDM to both synthetic and field data, and finally a discussion section.

**THEORY OF GDM**

I will first derive the equation for generalized diffraction stack migration, show how it can be interpreted as a dot product of the modeling operator and the data (Schuster, 2001), and then convert from shot-geophone coordinates to offset-midpoint coordinates. This last step gives the reverse-time migration formulae for any type of data, including COG data. It will then be easy to show how the usual Kirchhoff migration tricks can be applied to this operator. Later, I show how exact migration operators can be derived just from the data.

For weak scattering, the scattered data \( d(x_g, x_s) \) in the frequency domain can be explicitly written in terms of the Born approximation to the Lippmann-Schwinger equation (Stolt and Benson, 1986):

\[
\tilde{d}(x_g) = \int_{|x'|} \tilde{G}(x_g|\mathbf{x}') \tilde{G}(\mathbf{x}'|x_s) \tilde{m}(\mathbf{x}') d\mathbf{x}',
\]

(1)
Figure 1: (Left) Simple diffraction stack operator (dashed curve) and data, which only contains first-arrival scattering information. (Bottom) Generalized diffraction stack operator (dashed curves) which contains all events in the scattering model, including multiples, diffractions and reflections.
where $\tilde{G}(x'|x')$ is the Green’s function for the Helmholtz equation for a specified background medium with a source at $x'$ and a receiver at $x$; and $\tilde{m}(x')$ represents the reflectivity distribution perturbed from the background velocity. Here we have assumed an impulsive source wavelet.

Migration of seismic data can be carried out by applying the adjoint (Claerbout, 1992) of the forward modeling operator to the data to give the migrated image. The migration equation can be written as

$$m_{\text{mig}}(x') = \int_{\text{data space}} \tilde{G}^*(x'|x_s)|\tilde{G}^*(x_g|x')\tilde{d}(x_g)|dx_g,$$

(2)

where the $*$ denotes complex conjugation and the integration is over the data-space geophone variable denoted by $x_g$. Taking the complex conjugate of this equation gives

$$m_{\text{mig}}(x')^* = \int_{\text{data space}} \tilde{G}(x'|x_s)|\tilde{G}^*(x_g|x')\tilde{d}(x_g)|^*dx_g.$$

(3)

Summing equations 3 or 2 over all frequencies will give the same result because the migration image is purely real.

Summing over frequencies gives the formula for reverse-time migration (McMechan, 1983) in the space-time domain at zero lag:

$$m_{\text{mig}}(x') = \int \int g(x', t'|x_s, 0) \otimes_{t'=0} g^{\text{adj}}(x_g, t_g|x', 0) * d(x_g, t_g)|dx_g,$$

(4)

where the $d(x_g, t)$ term represents the data in the space-time domain and the bracketed term $g^{\text{adj}}(x_g, t_g|x', 0) * d(x_g, t_g)$ propagates backwards in time the trace energy at $x_g$ to the subsurface at $x'$. Also, $g(x', t|x_s, 0)$ propagates the energy at the source point $x_s$ to the subsurface and is then zero-lag correlated with the backpropagated trace energy. Traditional reverse-time migration uses a finite-difference method to solve the acoustic wave equation, where the point sources are at the traces located on the surface; the traces act as the time histories for backpropagating sources at the geophone locations.

A different implementation of reverse-time migration can be obtained by left shifting the brackets in equation 3 and summing over all frequencies to get

$$m_{\text{mig}}(x') = \int \int [g(x_g, t|x', 0) * g(x', t|x_s, 0)] \otimes_{t=0} d(x_g, t)|dx_g,$$

(5)

That is, the bracketed term

$$F(x_s, x', x_g, t) = g(x_g, t|x', 0) * g(x', t|x_s, 0),$$

(6)

in equation 5 acts as a focusing kernel so that reflection energy recorded at $x_g$ (for a source at $x_s$) is refocused back to $x'$. $F(x_s, x', x_g, t)$ can also be interpreted as the Born modeling operator for a point scatterer at $x'$, with a Green’s functions for an arbitrary background velocity. This leads to the important observation:

The GDM focusing kernel $g(x_g, t|x', 0) * g(x', t|x_s, 0)$ is the same as the Born point-scatterer modeling operator, where the point scatterer is located at the migration image point $x'$.

It is also noticed that the trace integration combined with the zero-lag correlation between the migration kernel and the data is nothing more than a dot product between the shot gathers and the appropriate migration kernels.
To see this, take the case of a single scatterer in a 2-layered medium, the diffraction stack migration kernel \( F(\mathbf{x}_g, \mathbf{x}_g', t) \approx \delta(\tau_{gr'} + \tau_{sr'} - t) \) has support along a hyperbola in data space coordinates \((\mathbf{x}_g, t)\). Here, \( \tau_{gr'} + \tau_{sr'} \) is the propagation time from the source point to the image point and back up to the geophone. Figure 2a depicts the graph of the focusing kernel at \( \mathbf{x}_g \) for a complex medium where there are several first arrivals at the subsurface scatterer point \( \mathbf{x}' \). There are several hyperbolic curves over which the data are summed over to give the migrated image at \( \mathbf{x}' \). This is equivalent to a dot product (in data space coordinates) of the data and the focusing (or point-scatterer modeling) operator.

The GDM image at \( \mathbf{x}' \) is formed as a dot product (in data space coordinates) between the data and the focusing operator.

Figure 1 illustrates this principle for a simple focusing operator (top illustration) and a general focusing operator that takes into account all events (bottom illustration).

Computing Focusing Kernels

The focusing kernel in equation 6 can be computed for a CDP experiment by one of two ways.

1. Place a source point on the surface at \( \mathbf{x}_s \), and solve for the field everywhere by a finite-difference method to get \( g(\mathbf{x}', t|\mathbf{x}_s, 0) \). Reciprocity says that \( g(\mathbf{x}_s, t|\mathbf{x}', 0) = g(\mathbf{x}', t|\mathbf{x}_s, 0) \), and if we replace \( \mathbf{x}_s \rightarrow \mathbf{x}_g \) then this gives \( g(\mathbf{x}_g, t|\mathbf{x}', 0) \). Thus, \( g(\mathbf{x}_g, t|\mathbf{x}', 0) \) can be convolved with \( g(\mathbf{x}', t|\mathbf{x}_s, 0) \) to give the focusing kernel in equation 6. See Figure 2.

2. Place a point source at depth \( \mathbf{x}' \) and solve for the field everywhere to get \( g(\mathbf{x}_g, t|\mathbf{x}', 0) \). Reciprocity says that \( g(\mathbf{x}_g, t|\mathbf{x}', 0) = g(\mathbf{x}', t|\mathbf{x}_g, 0) \), and letting \( \mathbf{x}_g \rightarrow \mathbf{x}_s \) yields \( g(\mathbf{x}', t|\mathbf{x}_s, 0) \). The focusing kernel at \( \mathbf{x}' \) can now be computed by equation 6.

Schuster (2001) and Zhou (2002) demonstrate how to efficiently compute these focusing operators by finite differencing along the leading portion of the wavefront.

GDM of Common Offset gathers

Although partial-migration methods such as phase-shift migration can be used to migrate COGs (Boland and Landro, 2001; de Bruin et al., 1990; Sava, 2001) it is not widely known how to implement COG migration with an RTM method (Paul Fowler, pers. comm.). To do this, simply apply the coordinate transformation \( x_g = x_o + x_m \) and \( x_s = -x_o + x_m \) (where \( x_m \) and \( x_o \) are the midpoint and half-offset coordinates, respectively) to give the RTM equations for COGs:

\[
m_{mig}(\mathbf{x}') = 2 \int \int [g(\mathbf{x}_o + x_m, t|\mathbf{x}', 0) * g(\mathbf{x}', t|\mathbf{x}_o + x_m, 0)] \odot d(\mathbf{x}_g', -t)|_{t=0} dx_m dx_o.
\]  

(7)

Here I started with an integral that integrated over both shot and receiver coordinates. The interpretation of this COG migration formula is given in Figure 3. Here, the source modeling kernel \( g(\mathbf{x}', t|\mathbf{x}_1, 0) \) is convolved with the scattering kernel \( g(x_2, t|\mathbf{x}', 0) \) to give the migration operator trace at midpoint \( x_m = (x_2 + x_1)/2 \) for the offset \( x_o = (x_1 - x_2)/2 \) (see Figure 3b). This procedure is repeated for all of the midpoint values at this offset value, and the ensemble of migration operator traces is the migration operator for the COG at offset \( x_o \). This procedure can be repeated for any offset value.
Figure 2: Focusing kernel decomposed into two modeling kernels. (a). Graph of focusing kernel $g(x'_s|x_s) * g(x'_g|x'_s)$ plotted against $x'_s$ and time for a fixed image point at $x'$ in a complex scattering model where there are several direct arrivals. Time variable has been suppressed. The source modeling kernel, i.e. $g(x'_s|x_s)$, is shown by the single trace at $x'$ in (b), and the scattering part $g(x'_g|x'_s)$ at $x_g$ is shown by the traces in (c). Convolving the trace in (b) with the traces in (c) gives the traces shown in (a). The migration kernel can also be computed by placing a source at $x'$ and use a finite-difference method to compute the shot gather traces on the surface. Choosing a source trace at the surface and convolving it with its neighbors gives the focusing kernel in (a).
Figure 3: Modeling kernel decomposed into two COG migration kernels. (a). Graph of modeling kernel \( g(x_1|x') \) for a buried source and receivers on the surface. The COG migration kernel for offset 1 and midpoint 1.5 \( g(x_1|x') \ast g(x_2|x') \) can be obtained by taking the trace at 1 and convolving it with the trace at 2 to give the trace shown in (b). Similarly, the COG migration kernel for offset 1 and midpoint 3.5 can be obtained by convolving the trace at 3 with that at 4 to give the trace shown in (c). In fact, all COG migration kernels with any offset and any midpoint position can be obtained by convolving the appropriate traces from the single shot gather in (a)!
NUMERICAL RESULTS

Wave equation wavefront migration is now applied to synthetic data associated with the layered model shown at the top of Figure 4. The following steps describe the procedure for generating these results.

1. The synthetic data traces on the free surface were generated by a 2-2 FD solution to the acoustic wave equation.

2. A shot point was placed at different depths in the velocity model and traces were computed on the surface. One of these surface traces was selected as a virtual source trace and convolved with the neighboring traces to get the focusing kernel for that shot gather. COG focusing kernels were computed in the same way, except only focusing kernels with a constant offset were computed.

3. Equation 5 was used to compute the migration image.

GDM of COG’s

Common offset gathers were extracted from the synthetic CSG’s (see bottom panel in Figure 4), the focusing kernels for different depth levels were computed by placing a source at these depths and finding the FD solutions at the surface. The focusing kernels were then computed from these solutions by simple convolutions of a surface trace with its neighbors, as previously described. The COG was migrated to different depth levels using the COG focusing kernels. The results in Figure 5 show that the layer boundaries are correctly imaged, including the layers below the 1st later with a velocity gradient. This result is exactly equivalent to RTM of the data.

Filtering Focusing Kernels

If the maximum energy traveltimes and amplitudes are picked from the up- and downgoing modeling kernels then a Kirchhoff-like migration can be applied to the data. However, no high frequency approximation is needed as in standard Kirchhoff migration. Moreover, anti-aliasing is automatically employed if the data are slant stacked (Sun and Schuster, 2002; Sun, 2001) to reveal the specular reflection angles and the focusing operator is muted to admit only these angles of incidence. See Zhou (2002) for an example.

Exact DSM Operators from VSP Data

The ability to obtain a focusing gather by simple convolution of surface traces generated by a shot at depth suggests that VSP data can be used to obtain the exact DSM focusing kernel. No velocity model or forward modeling operations are needed. The data-derived focusing kernel contains exactly all of the multiples, converted waves, and diffractions that mother earth can provide. This might be useful if VSP data have been collected below complex salt bodies that defy conventional efforts to image below the salt. The VSP data can be resorted and reciprocity applied to give virtual RVSP traces, where the source is at depth and the traces are recorded on the surface. This is the shot gather needed to get the exact DSM operator for surface data and image points along the well. DSM operators for image points away from the well can, in principle, be extrapolated by a wave-equation method if an estimate of the velocity model is known. The data-driven DSM kernels can be vetted to see how they can be filtered to optimize imaging quality. For example, eliminate some hyperbola-like curves related to multiples.

Figure 7 shows an example, where a surface-recorded CSG with a source at the surface is shown in the top figure (direct wave muted). The goal is to migrate these traces to different depths using just the RVSP data obtained at a nearby well (not shown). Here the RVSP traces in a shot gather
Figure 4: (Top) Velocity model with gradient in top layer and (bottom) CSG data.
Figure 5: (Top) COG data, (middle) GDM focusing operator for these data with the image point at depth 83 m, (bottom) migration image. Layer boundaries in previous figure have been correctly imaged.
Figure 6: (Left) COG focusing kernels at different image point depths, (right) same as left side except aperture width truncated to prevent aliasing. Aperture width increases with depth and the constant offset is 750 m.
are convolved with the master RVSP trace at the CSG shot position, to get the DSM focusing operator at depth (see middle figure). Focusing kernels at different depths are applied to the CSG data (top figure) to give the migration image shown in the bottom panel.

Figures 8-9 give the results of computing focusing operators from RVSP data collected by Exxon in Friendswood, Texas. Events in the focusing operators can be picked to give Kirchhoff focusing operators for primary or multiple reflections, as shown in Figure 9. These operators can be used to distinguish primaries from multiples in CMP velocity analysis, for least squares migration filtering methods, and for subtraction of multiples. These focusing operators can, in principle, be extrapolated to image points away from the well by either comparison/interpolation to neighboring CSG’s or by wavefield extrapolation.

**SUMMARY AND DISCUSSION**

I have shown that RTM is equivalent to generalized diffraction stack migration. The generalized migration image at a point $\vec{x}'$ is obtained by taking the dot product of the appropriate focusing operator with the data. This is a generalization of simple diffraction stack migration which sums the data over the appropriate hyperbola (i.e., simple focusing operator).

Some immediate uses of GDM include the following.

1. Deterministic filtering of the GDM focusing kernels can potentially be used to mitigate multiples, eliminate aliasing artifacts and reduce noise in the RTM section.

2. A reverse-time common-offset migration operator is obtained without any approximation. This operator can be applied to common-offset gathers for any media, including those that induce turning waves.

3. Target-oriented RTM has been developed with the suggestion of Yi Luo.

4. A more efficient RTM method is developed that only uses first-arrival information to back-propagate arrivals. I called this wavefront wave-equation migration in a previous report.

5. GDM can be computed by any modeling method, including the reflectivity method.

6. Anti-aliasing, obliquity factors, and depth-dependent truncation of data aperture tricks can be applied to the GDM operator. In fact, a generalization of Gaussian-Beam and wavepath migration can be formulated.

7. The RTM reformulation leads to a theory for construction of the *exact* reverse-time migration operator from VSP data alone. No modeling computations or velocity model are needed, except simple convolutions of the VSP traces. This could be important in imaging beneath salt domes where VSP data is available. In addition, migration operators for multiples can be derived from the VSP data as well. Focusing operators for multiples can easily be obtained from these data.

The biggest obstacle in implementing GDM is the memory and computation expense of the focusing operators. Thus, full blown GDM should be restricted for now to 2-D data, but reduced forms of it such as wavefront migration (Zhou, 2002) might be feasible in a target-oriented mode. Efficiency tricks such as variable grid finite-difference methods, phase encoded simulations and the rapidly growing capability and efficiency of cluster computing may someday make GDM of 3-D data a reality.

Finally, there might be some connection with this work and that of Berkhout and Verschuur (2001) but I need to read and understand their paper to make that connection.
Figure 7: (Top) Shot gather generated by a source at the surface, except direct wave removed for migration purposes. Layered model in Figure 4. (Middle) CSG focusing operator obtained by convolving traces in a RVSP shot gather generated by a source at depth and geophones on the surface. The master trace was at the same position as the source that generated the CSG. (Bottom) Migration image obtained by applying focusing operators at different depths to the top CSG. Focusing operator for multiples can be easily obtained by muting the direct arrival in the RVSP shot gather, and use the master trace for convolution. This will be the exact full multiple DSM operator. Partial multiple focusing operators can obtained by more delicate muting of arrivals in the RVSP shot gather.
Figure 8: (Top) Shot gather generated by a source at depth from an RVSP experiment in Friendswood, Texas (data courtesy of Exxon). (Bottom) DSM focusing operator for a surface CSG gather with a surface shot at x=24 m. Focusing operator obtained by convolving 2nd trace at far left in top panel with all other traces.
Figure 9: (Top) Kirchhoff primary reflection focusing operator obtained by picking first arrival from the bottom panel in previous figure. (Bottom) Same as top, except Kirchhoff multiple focusing operator obtained by picking arrival of an interbed multiple in the CSG focusing panel. Interbed multiples are easy to identify in RVSP data.
REFERENCES


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