Interferometric Seismic Imaging

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ABSTRACT

Claerbout’s daylight imaging concept is generalized to the general theory of interferometric seismic imaging (II). Interferometric seismic imaging is defined to be any algorithm that inverts crosscorrelated seismic data for the reflectivity or source distribution. As examples, we show that II can be used to image reflectivity distributions by migrating ghost reflections in passive seismic data, and II generalizes the receiver-function imaging method used by seismologists. Interferometric seismic imaging can also be used to migrate free-surface and peg-leg multiples in CDP data and image source distributions from passive seismic data. Both synthetic and field data examples are used to illustrate the different possibilities of II. The key advantage of II is that it can be used to image source locations or reflectivity distributions from passive seismic data where the source position or wavelet is unknown. In some cases it can mitigate defocusing errors due to statics or an incorrect migration velocity. The main drawback with II is that severe migration artifacts can be created by partial focusing of virtual multiples.

INTRODUCTION

Passive seismic imaging can be divided into two categories: firstly, attempts to image the spatial locations of passive seismic sources themselves, and secondly, attempts to image the subsurface reflectivity that is illuminated by passive seismic energy.

Passive Seismic Source Imaging

Passive seismic source imaging has the unique potential to provide direct measurements of subsurface permeability (e.g. Shapiro et al., 1999). Fluid flow causes fracturing; you image the fracturing; therefore, you are imaging the fluid flow. This,
along with the growth of (both surface and borehole) time-lapse seismic, has led to the drive towards the “electric oilfield” permanently instrumented and continually monitoring itself (Jack and Thomsen, 1999).

To date, however, most of the published case studies of microseismic fracture imaging rely on earthquake-style hypocentral event triangulation. For example, Maxwell et al. (1998) describes the successful application of such technology to the Ekofisk field in the North Sea. These approach require automated event picking algorithms, and may run into problems if microseismic events are not localized in time.

**Reflectivity Imaging with Passive Seismic Energy**

Baskir and Weller (1975) describe possibly the first published attempt to use passive seismic energy to image subsurface reflectivity. They briefly describe crosscorrelating long seismic records to produce correlograms that could be processed, stacked and displayed as conventional seismic data. Unfortunately their field tests seem to have been inconclusive.

Dating from about the same time, an exercise in Claerbout’s first book (1976) asks the reader to prove that the temporal autocorrelation of a transmission seismogram for a layered model with a source deep underground is equivalent to a reflection seismogram. This may have inspired his conjecture that by crosscorrelating two passive traces, we can create the seismogram that would be computed at one of the locations if there was a source at the other. Cole (1975) attempted to verify this conjecture with data collected using a 4000 channel 2-D field array on Stanford campus. Unfortunately, again, possibly due to the short (20 minute) records or bad coupling between the geophones and the dry California soil, his results were inconclusive.

Following Cole’s work, Rickett and Claerbout (1996) generated synthetic data with the phase-shift method. Their earth reflectivity models consisted of (both flat and dipping) planar layers and point diffractors embedded in a $v(z)$ velocity function, and illuminated by random plane waves from below. They generated both *pseudo shot gathers* (by crosscorrelating one passive trace with many others nearby), and *pseudo zero-offset sections* (by autocorrelating many traces). In these crosscorrelated domains, the kinematics for both point diffractors and planar reflectors, were identical to those predicted for real shot gathers and zero-offset sections. Rickett and Claerbout (1999) then experimented with moving the passive source location close to the receivers and reflectors, and included modeling with a $v(x, z)$ velocity model. He observed that these changes did indeed affect the kinematics of the correlograms; however, changes were small, and would probably not cause the method to fail in most situations.

The idea that a reflection seismogram could be created by crosscorrelating two pas-
sive seismic records was rediscovered independently by the helioseismologists (Duvall et al., 1994), who created time-distance curves by cross-correlating passive solar dopplergrams recorded by the Michelson Doppler Imager (Scherrer et al., 1995). Point-to-point traveltimes derived from these time-distance curves could then be used in a range of helioseismic applications (e.g. Giles et al., 1997 and Kosovichev, 1999). If helioseismic time-distance curves are averaged spatially, the result is equivalent to a multi-dimensional autocorrelation. Rickett and Claerbout (2000) demonstrated that multidimensional spectral factorization provides spatially averaged time-distance curves with more resolution than those calculated by autocorrelation. Their demonstration was restricted to layered models with no lateral velocity variation.

In 1990 Katz received a patent for applying Claerbout’s 1-D autocorrelogram imaging method to VSP data. Using VSP data obtained from a rotating drill bit, Katz showed that 1-D images of the earth’s reflectivity could be obtained by autocorrelating the traces recorded on the free-surface. Later, Schuster et al. (1997, 2002) and Yu et al. (2002) generalized the Katz algorithm from 1-D imaging to the theory of multidimensional migration of autocorrelograms.

**Extending Daylight Imaging: Interferometric Imaging**

Here we present the mathematical framework for imaging crosscorrelated data, i.e., interferometric imaging, for arbitrary reflectivity or source distributions. We show that interferometric imaging extends the daylight imaging concept to any number or distribution of sources and to arbitrary reflectivity distributions. Moreover, it offers new imaging opportunities, such as a very simple means to migrate multiples in data, migrate transmitted waves, or locate unknown source locations from daylight data. Simply put, interferometric imaging can be described as crosscorrelation migration (Schuster, 1999; Schuster and Rickett, 2000), or an extended form of autocorrelation migration (Schuster et al., 1997).

Instead of exploiting the entire phase of arrivals, interferometric imaging exploits the phase difference between different arrivals. These phase differences can reveal subtle variations between the arrivals, which can be indicative of subtle changes in the medium properties. For example, sunlight on an oil slick at sea can produce a rainbow of interference patterns: reflections from the top of the oil slick interfere with those from its bottom to reinforce at certain light colors and thicknesses of the oil slick. The common raypath of the top and bottom reflections have equal and opposite phase that can cancel one another, and the phase difference we see accounts for the phase change along the transit path in the oil. Similarly, seismologists can construct interferometric data by crosscorrelating trace A with trace B. In this way we can exploit the phase difference between a certain arrival in trace A with certain arrivals
in trace B. We will now generalize this interferometric imaging idea so that it extends the daylight imaging idea of Claerbout and his students to arbitrary reflectivity and source distributions. We also show how II can be used to image free-surface multiples or peg-leg multiples from CDP data, generalize the receiver-function imaging of PS transmitted waves used by seismologists, and image source locations of unknown sources at depth.

INTERFEROMETRY

For the last century, optical interferometry has played an extremely important role in advancing the fields of physics, astronomy and engineering. The key idea is that a light beam is used to sample the properties of an object or medium, and is combined with a reference beam. The resulting interference pattern is sometimes called an interferogram, and magnifies subtle optical properties of the object. Subtle changes are magnified because the interferogram highlights differences in the phases between the reference and sampling beams.

Optical Interferometry

Figure 1 depicts two interfering beams, where a laser beam illuminates the lower portion of the lens and the interferogram is recorded above the lens. The interferogram is the result of the interference between the reference direct wave (sA), denoted by \( \tilde{d}_A = e^{i\omega\tau_{sA}} \) and the wave reflected within the lens (sArB) denoted by \( \tilde{d}_B = R^2 e^{i\omega(\tau_{sA} + \tau_{Ar} + \tau_{rB})} \):

\[
\tilde{d}_A = e^{i\omega\tau_{sA}}; \quad \tilde{d}_B = R^2 e^{i\omega(\tau_{sA} + \tau_{Ar} + \tau_{rB})}.
\]  

(1)

Here, \( \tau_{ij} \) is the propagation time along the path \( ij \), \( R \) is the small reflection coefficient associated with the glass-air interface and \( \omega \) is the angular frequency of the optical wave. Dark lines in the interferogram denote the zones where the reflection and direct beams are out of phase, and the in-phase zones depict coherent interference. Changes in the lens thickness will result in phase changes between the direct and reflected arrivals, so producing the ring-like interferogram. Deviations from a ring-like pattern suggest anomalies in the lens geometry.

Mathematically, the interferogram is the intensity of the summed direct and reflected waves:

\[
I = (\tilde{d}_A + \tilde{d}_B)(\tilde{d}_A + \tilde{d}_B)^* = 1 + 2R^2 \cos(\omega[\tau_{Ar} + \tau_{rB}]) + R^4,
\]  

(2)

where the intensity pattern \( I \) is controlled by the phase \( \omega[\tau_{Ar} + \tau_{rB}] \) along the reflected portion of the raypath. Note the important observation: the intensity or ring-like
Seismic Interferometry

Seismic interferometry is similar to optical interferometry, except seismic waves are used instead of an optical beam and the interferogram is obtained by crosscorrelating neighboring traces. As an example, Figure 2a illustrates the case of a harmonic source at some unknown depth and with some unknown source wavelet. The goal is to estimate the reflectivity distribution from the seismic traces recorded at A and B, and given in equation 1. Instead of summing the two traces together, the conjugate of the trace spectrum at A is multiplied with the trace spectrum at B to give

$$\tilde{\Phi}_{AB} = \tilde{d}_A^* \cdot \tilde{d}_B = Re^{i\omega(\tau_A + \tau_B)} + o.t.,$$

(3)

where $\tilde{\Phi}_{AB}$ denotes the product spectrum, the exponential term represents the correlation of the direct wave at A with the ghost reflection recorded at B, and $o.t.$ denotes other terms such as the direct-direct or reflected-reflected wave correlations.
Just like the laser intensity function in equation 2, \( \tilde{\Phi}_{AB} \) is a function of the phase \( \omega(\tau_{Ar} + \tau_{rB}) \) along the reflected portion of the raypath. Thus, changes in reflector geometry lead to changes in the correlated data \( \tilde{\Phi}_{AB} \). Later, we will see how to recover the reflector geometry by applying the migration kernel \( e^{-i\omega(\tau_{Ar} + \tau_{rB})} \) to \( \tilde{\Phi}_{AB} \).

In the time domain, the above product becomes the cross-correlation between the trace at \( A \) with that at \( B \). For this paper, it will be understood that crosscorrelation of traces will be referred to as either a product of spectrums in the frequency domain or trace correlation in the time domain.

**INTERFEROMETRIC SEISMIC IMAGING**

Interpretation of the raw interferograms in equation 3 for subsurface geology is too cumbersome. Instead, an image of the reflectivity distribution can be obtained by migrating the crosscorrelated traces, otherwise known as interferometric seismic imaging (II). Crosscorrelation migration is similar to standard migration in that an imaging condition is applied to the backprojected data, except in seismic interferometry the input data are the crosscorrelograms. Several examples of II will now be presented: imaging of the reflectivity distribution from data generated by sources below and on the free surface, and imaging of buried source locations.

**Ghost Reflection Crosscorrelogram Imaging with Buried Sources**

Figure 2a illustrates the case where a drill bit at depth radiates seismic energy that is recorded on the free surface. The source wavelet in the frequency domain is denoted as \( \tilde{W}_i(\omega) \), where \( i \) denotes the \( i \)th source wavelet to account for the fact that the drill bit can occupy widely separated positions in depth, each characterized by a different wavelet. The phase of the source wavelet at the \( i \)th position is random and assumed to be uncorrelated with the source wavelet at other positions, i.e., \( w_i(t) \otimes w_j(t) = 0 \) for \( i \neq j \).

The goal is to use ghost reflections to reveal the geological layering, despite the fact that the bit location is uncertain and the source wavelet is very ringy. This is a problem that can be solved by II: crosscorrelation tends to collapse ringy source wavelets to short duration and also eliminates the need to know the source location, as shown in the following steps.

1. The frequency-domain traces at positions \( B \) and \( A \) are given by (see Figure 2a)

\[
\begin{align*}
\bar{d}_B &= \tilde{W}_i(\omega) e^{i\omega \tau_{sB}} - \tilde{W}_i(\omega) Re^{i\omega(\tau_{sA} + \tau_{Ar} + \tau_{rB})} + o.t., \\
\bar{d}_A &= \tilde{W}_i(\omega) e^{i\omega \tau_{sA}} - \tilde{W}_i(\omega) Re^{i\omega(\tau_{sA} + \tau_{Ar} + \tau_{rA})} + o.t. \quad (4)
\end{align*}
\]
where the specular-ghost and direct-wave terms are explicitly written while all other terms such as primary reflections, multiples and diffractions. Geometrical spreading is harmlessly ignored and the angle-dependent reflection coefficient at the layer interface is taken to be $R$.

The ray $sArB$ is the specular raypath for the free-surface ghost reflection that begins at $s$ and terminates at $B$ as shown in Figure 2a. Similarly, $sA'r'A$ is the associated raypath for the specular ghost reflection that also begins at $s$ but terminates at $A'$. Here, $A'$ is the specular reflection location at the free surface.

2. Form the correlated data. Crosscorrelating trace $A$ with trace $B$ gives

$$\tilde{\Phi}(A, B) = \tilde{d}_A^* \cdot \tilde{d}_B$$

$$= |\tilde{W}(\omega)|^2 e^{i\omega(\tau_{sB} - \tau_{sA})} - R(e^{i\omega(\tau_{Ar} + \tau_{rB} + \tau_{sA} - \tau_{sA})} + e^{i\omega(\tau_{rB} - \tau_{sA} + \tau_{sA} - \tau_{sA})})$$

$$+ R^2 e^{-2i\omega(\tau_{Ar} + \tau_{A'r'B}) + 2e^{2i\omega(\tau_{sA} + \tau_{A'r'B})}] + \ldots}$$

$$= -|\tilde{W}(\omega)|^2 Re^{i\omega(\tau_{Ar} + \tau_{rB})} + o.t.,$$

(5)

where the $direct_A - direct_B$ correlation is of the most importance because it does not contain the unknown source-phase term $\omega\tau_{sA}$. In fact, $\tilde{\Phi}(A, B) \approx Re^{i\omega(\tau_{Ar} + \tau_{rB})}$ is kinematically equivalent to a shot gather of primary reflections with a source at $A$ and traces at $B$.

3. Migrate the crosscorrelated data $\tilde{\Phi}(A, B)$. The migration kernel should be tuned to annihilate the phase of the $ghost_B - direct_A$ correlation when the trial image point is at the reflector. Thus, multiplying $\tilde{\Phi}(A, B)$ by the ghost migration kernel

$$e^{-i\omega(\tau_{Ax} + \tau_{xB})}$$

(6)

and summing over all frequencies, virtual source positions $A$ and geophones $B$ yields the migration image $m(x)$ at $x$:

$$m(x) = \sum_{\omega} \sum_{A,B} \tilde{\Phi}(A, B)e^{-i\omega(\tau_{Ax} + \tau_{xB})},$$

$$= \sum_{A,B} \phi(A, B, \tau_{Ax} + \tau_{xB}),$$

(7)

where $\phi(A, B, t)$ is the temporal correlation between the traces at $A$ and $B$ with lag time $t$. When $x$ is coincident with the actual specular reflection point at $r$
a). Passive Seismic Imaging

\[ m(x) = e^{i \omega (\tau_{Ar} + \tau_{rB})} \]

b). CDP Multiple Imaging

\[ m(x) = e^{i \omega (\tau_{Ar} + \tau_{rB})} \]

c). Transmitted PS Imaging

\[ m(x) = e^{i \omega (\tau_{rA} - \tau_{rB})} \]

d). Source Location Imaging

\[ m(x) = e^{i \omega (\tau_{sA} - \tau_{sB})} \]

Figure 2: Crosscorrelation migration operators \( m(x)|_{x=r} \) tuned to different correlated events for the data \( d_A^* d_B \). (a). Buried source (such as a drill bit with unknown location) with a the direct wave at A and the ghost reflection at B. (b). Same as (a), except the source is at the surface and its location is known, such as in CDP data. (c). Correlated event is the transmitted P wave at B and the transmitted PS-wave at A. (d). Same as (c), except now the source location is sought and the two correlated events are the direct P waves at A and B.
then there will be annihilation of the ghost$^B - direct_A$ phase $\omega(\tau_A + \tau_B)$ in equation 5 to give maximum migration amplitude at $x = r$ for all $\omega$. The other terms (denoted as virtual multiples in Schuster et al., 1997) will be, hopefully, incoherently focused just like the migration of actual multiples by Kirchhoff migration. In summary, the ghost reflection is used to image the reflector, even though we do not know the source location or the source time history! The above equation is that of standard prestack diffraction-stack migration, except the input data are the crosscorrelograms.

The above methodology is applicable to any number of sources, any depth of source burial, and can image an arbitrary reflectivity distribution. For multiple sources contemporaneously excited, the success of this method demands that the source wavelet time histories be uncorrelated, e.g., a random time series.

One of the implicit assumptions is that the trace at $A$ is at the specular reflection point on the free surface for the $sArB$ raypath. For a high-frequency source at $s$, non-specular reflections do not significantly contribute to the imaging at $r$ as shown by stationary-phase analysis in the appendix.

**Five-Layer Synthetic Data Example.** A five-layer geologic model is used to test the crosscorrelogram migration method, and roughly represents the recording geometry and model for drill-bit data collected in a W. Texas experiment (see Yu et al., 2002). The top panel in Figure 3 shows the velocity model used for crosscorrelogram migration. The synthetic drill-bit source is moving horizontally at the depth of 1500 m, and the drill bit moved in the horizontal direction from 1650 m to 1940 m during the recording sessions. The data are recorded with a source interval of 5 m. The receivers were deployed on the surface over a lateral range of 4000 m, and the receiver interval is 20 m; there were a total of 39 common source gathers (CSG) recorded. The synthetic data were generated by computing the solution of the acoustic wave equation with the finite-difference method.

The middle panel in Figure 3 shows a typical common source gather. Besides primary reflections, there are free-surface related ghost waves and interbed multiples. The bottom panel in Figure 3 shows the crosscorrelograms computed from the middle panel shot gather.

Applying equation 7 to these data gives the migration images shown in Figure 4. The top panel shows that the crosscorrelogram migration image has spurious events caused by partial focusing of virtual multiples such as $direct - primary$ correlations. Using velocity filtering to separate the primary reflections in the input data, the false reflectors have mostly disappeared and the subsurface structure is well reconstructed as shown in Figure 4.
Figure 3: (Top) Velocity model (middle) shot gather and (bottom) crosscorrelograms using trace 80 as the master trace.
Figure 4: Crosscorrelogram migration images in time domain (a). with primary and ghost reflections; (b). without primary reflections. Here the migration operator is tuned to the correlation between the direct wave and the ghost reflection. The arrows indicate the actual reflector locations.
Ghost Reflection Autocorrelogram Imaging with Buried Sources

We will now assume that source position at depth is known, which will allow us to reduce computational expense by only having to migrate autocorrelograms. In addition, migration of autocorrelograms reduces the focusing errors due to both migration velocity errors and static effects (Sheley and Schuster, 2000; Yu et al., 2002).

To understand these last statements, include the specular primary reflection term in equation 4 so that

$$\tilde{d}_B = \tilde{W}_i(\omega) [e^{i\omega \tau_{sB}} + Re^{i\omega (\tau_{sA'} + \tau_{rB})} - Re^{i\omega (\tau_{sA} + \tau_{rA} + \tau_{rB})}] + ...$$

(8)

The autocorrelation function for the trace at $B$ becomes

$$\tilde{\Phi}(B, B) = |\tilde{W}(\omega)|^2 [1 + 2R^2 + R(e^{i\omega (\tau_{sB} - \tau_{sA} - \tau_{rB})}) + R(e^{i\omega (\tau_{sA} - \tau_{Ax} - \tau_{rB})})] + o.t.. (primary_B - direct_B)$$

(9)

$$\tilde{\Phi}(B, B) = |\tilde{W}(\omega)|^2 [1 + 2R^2 + R(e^{i\omega (\tau_{sB} - \tau_{sA} - \tau_{Ax} - \tau_{rB})}) + R(e^{i\omega (\tau_{sA} - \tau_{Ax} - \tau_{rB})})] + o.t.. (ghost_B - direct_B).$$

If the source position at $s$ and the migration velocity are known then the specular rays for the ghost reflections can easily be computed to give the traveltime fields $\tau_{sA'} + \tau_{Ax}$ for all subsurface points $x$ and their specular free-surface reflection points $A'$. Note, $A'$ depends on the source point location and the trial image point location $x$. This differs from the previous section where the source position was unknown.

The migration kernel $e^{-i\omega (\tau_{sB} - \tau_{Ax} - \tau_{rB})}$ focuses $primary_B - direct_B$ correlations to the layer interface. The resulting migration image will be denoted as the primary autocorrelation image $m(x)_{primary}$:

$$m(x)_{primary} = \sum_{\omega} \sum_{B} e^{-i\omega (\tau_{sB} - \tau_{Ax} - \tau_{rB})} \tilde{\Phi}(B, B).$$

(10)

In addition, the $ghost_B - direct_B$ correlations can be focused to the layer interface by applying the migration kernel $e^{-i\omega (\tau_{sB} - \tau_{Ax} - \tau_{rB})}$ to yield the ghost autocorrelation image.

$$m(x)_{ghost} = \sum_{\omega} \sum_{B} e^{-i\omega (\tau_{sB} - \tau_{Ax} - \tau_{rA} - \tau_{rB})} \tilde{\Phi}(B, B).$$

(11)

An advantage of knowing the source location is that the autocorrelation migration equation only sums once over the geophone positions compared to the double nested loop over geophone positions in the crosscorrelation migration in equation 4. This results in less computation time and fewer migration artifacts. The additional loop
over geophone index $A$ in equation 6 is needed in order to involve the trace at the specular reflection location on the free surface (see Figure 2a), which is unknown for a source buried at an unknown location.

Figure 5 depicts the ghost autocorrelation migration and blended autocorrelation migration images for the five-layer model. The blended image was obtained by computing the blended product of $m(x)_{\text{ghost}}$ and $m(x)_{\text{primary}}$. Note, the blended image is almost free of migration artifacts.

W. Texas Drill-Bit Data. Drill-bit seismic data were recorded with ten three-component receivers in W. Texas by Union Pacific Resources Co. (UPRC); the receivers were equi-spaced between 822 m and 2100 m from the drill rig as shown in Figure 6. The data are recorded on the earth’s free surface while a tri-cone drill-bit and down-hole motor were used to drill along a horizontal trajectory at a depth of 2800 m in the Austin Chalk formation. There are about 609 shot gathers, each with a recording length of about 20 seconds with a sample interval of 2 ms. Because the seismic data were distorted by strong noise, the data were preprocessed as described by Yu et al. (2001).

The inset in the bottom panel of Figure 7 shows the joint autocorrelogram migration result using both primary and ghost reflections, where the trace interval is about 3.038 m. The top panel shows the primary autocorrelation migration image. In comparison, it can be seen that joint autocorrelogram migration generates a look-ahead image with less interference.

Free-Surface Multiple Imaging with CDP Data.

Now we will show how interferometric imaging can be used to migrate free-surface multiples in CDP data. In comparison to the Delft method (Berkhout and Verschuur, 1999 and 2000) of autoconvolving traces and subtracting the computed multiples from the original traces, we will crosscorrelate the data to generate shifted pseudo-replicas of multiples and migrate these multiples. The multiple migration image is then combined with the primary reflection migration section to determine the common reflector locations. Delft’s strategy to attack interbed multiples can be followed as well, except with interferometry the interbed multiples are incorporated into the migration section.

Placing the source at the surface gives rise to the diagram in Figure 2b. Here the free-surface multiple recorded at B and the primary reflection recorded at A are explicitly represented by

$$
\tilde{d}_B = R^2 \tilde{W}(\omega) e^{i \omega (\tau_{sr} + \tau_{rA} + \tau_{Ar} + \tau_{rB})} + \text{o.t.}; \quad \tilde{d}_A = -R \tilde{W}(\omega) e^{i \omega (\tau_{sr} + \tau_{rA})} + \text{o.t.} \quad (12)
$$
Figure 5: (Top) Ghost autocorrelation migration image and blended image using both the primary and ghost autocorrelation images for the five-layer model.
where only the terms of interest are explicitly included in the equation. The cross-correlation of these two traces annihilates the common phase terms in the exponents to give

$$ d_A^* \cdot d_B = R^3|\tilde{W}(\omega)|^2 e^{i\omega(\tau_{A_x}+\tau_{r_B})} + ..., \quad (13) $$

where the phase term in this equation suggest kinematics equivalent to a primary reflection generated by a source at A and a receiver at B. This is similar to the case of a buried source except the strength of the correlation has been reduced in equation 6 from $R$ to $R^3$!

The obvious migration kernel for the correlated data is given by $m(x) = e^{-i\omega(\tau_{A_x}+\tau_{r_B})}$ so the migration equation is exactly the same as equation 7. However, a major problem with $\Pi$ is that the migration kernel is tuned to a correlation with a weak strength of $R^3$. This weak correlation competes with stronger correlations, such as primary with primary correlations that are $R^2$ strength that can inadvertently be tuned to the multiple migration kernel (see Sheng et al., 2001).

**SEG/EAGE Salt Model Data.** The SEG/EAGE salt model is chosen to test the effectiveness of migrating multiples in CDP data. The top illustration in Figure 8 shows profile A-A from the SEG/EAEG salt model. The model used is 17120 meters by 4000 meters, with a trace interval of 27 meters, and a trace recording length of 5 seconds. There are 320 shot gathers, each with 176 traces. The middle and bottom
Figure 7: (Top) Ghost autocorrelation migration image and blended image using both the primary and ghost autocorrelation images for the W. Texas data.
images show the prestack Kirchhoff and crosscorrelogram migration images, respectively. As expected, the crosscorrelation images contain more artifacts because the virtual multiples are migrated to incorrect locations. Similarly, but not to the same severity, the Kirchhoff image also contains incorrectly imaged multiples. The arrows in the Kirchhoff image point towards the incorrect imaging of multiples.

Both the crosscorrelation and Kirchhoff images both show the events correctly imaged at the actual reflector positions. Therefore, a weight $w_i$ can be computed that grades the similarity between the Kirchhoff $KM(i)$ and crosscorrelation $CCM(i)$ images in a local window centered at the $ith$ pixel. The weight $w_i$ is computed by correlating the Kirchhoff migration traces with the corresponding crosscorrelation migration traces in a small window for each shot gather. In practice, the window is 40 traces wide and 20 sample points tall. The final merged image for a migrated shot gather can be obtained by

$$Merged(i) = w_i KM(i),$$

and the composite merged imaged is computed by summing the merged images for all shot gathers.

The merged image obtained by applying the above procedure to the CCM and KM images is given in Figure 9. It can be seen that at the left part of the image the true reflectors are enhanced and the artifacts caused by the free-surface multiples are attenuated. Below the salt body, it does not show much improvement which might be due to the Kirchhoff migration method itself.

**Teleseismic Receiver Function Imaging**

Seismologists use converted PS transmission waves to image the geometry of a layer interface, often the Moho (Langston, 1977; Bostosck and Ronenay, 1999; Sheley and Schuster, 2000). For a recorded teleseism, they crosscorrelate the vertical component with the horizontal component, where the largest correlation amplitude is presumed to be the converted PS transmitted wave at the Moho. The lag time of this PS correlation is related to the depth of the Moho if the P/S velocity ratio is known. Figure 2c shows the ray diagram for transmitted waves that are converted at the interface. It can be seen from this diagram that the crosscorrelation of trace A with B will annihilate the common phase term along the ray $sr$, so that the PS transmission migration kernel is shown in the figure.

As an example of imaging the crust with teleseismic ghost reflections (Sheng et al., 2001), elastic seismograms from a plane P-wave source were computed by a 2-D finite-difference solution to the elastic wave equation. Figure 13 shows these seismograms with a source incidence angle of 10 degrees. Direct and surface reflected phases are
Figure 8: (Top) SEG/EAGE salt model, (middle) Kirchhoff prestack migration image, (bottom) crosscorrelation migration image.
seen in the data where the crustal model is a 4-layer model shown by the white lines in Figure 11. The model is modified from an E-W cross section across northern Utah by Loeb (1986); the trough in the third layer boundary was added for testing purposes. The source time history was modeled as a Ricker wavelet with a peak frequency of 0.6 Hz and a bandwidth of approximately 0.2 to 1.2 Hz. The station spacing is 1 km.

Figure 11 shows the reflector image of the 4-layer crustal model obtained by migrating ghost reflections (equation 7) in a synthetic teleseismic record. The result of the correlogram migration is that the upper two interfaces are correctly imaged, while the third one is contaminated by a second-order ghost. It is expected that data from different incidence angles will suppress this source of coherent noise.

Source Location Imaging

Sometimes it is desirable to locate the unknown position of a seismic source, such as in the case of a hydro-frac test where the induced fracture location indicates the fluid pathway. In this case, we can use the $\text{direct}_A - \text{direct}_B$ correlation in equation 5 to find the unknown source position $s$ in Figure 2d. That is, apply the migration kernel $e^{-i\omega(\tau_s B - \tau_s A)}$ to the data $\Phi(A, B)$ to get the migration image of the source.
Figure 10: (Top) Ray diagram for an earthquake generating a ghost reflection from the free surface. (Bottom) Seismograms generated by a teleseismic plane P-wave with an incident angle of 10 degrees. The direct and ghost reflections are prominent where the crustal model is the 4-layer crustal model for Utah.
Figure 11: Image of interfaces after cross-correlogram migration of the surface reflected P waves in the cross-correlated data in the previous figure. The first and second interfaces (white solid lines) are correctly imaged, but the deepest interface is obscured by spurious events in the correlated records that should be reduced by stacking teleseismic event records. Interface model (white lines) is similar to that of the crust along an east-west profile in central Utah.
locations:
\[ m(x) = \sum_{A,B} \sum_{\omega} \Phi(A, B) e^{-i\omega(\tau_{sB} - \tau_{sA})}. \]  

(15)

Single scattering synthetic data generated by a ray tracing method will be used to test this concept.

Figure 12a shows synthetic data generated for a point exploder centered 1050 m below a 2100 m wide array. There are 70 geophones in the array with a geophone spacing of 30 m. The traces are computed for 1 second of duration with a 30 Hz Ricker wavelet source. The point scatterer responses of the diffraction stack migration and the crosscorrelation migration are shown in Figures 12b and 12c, respectively. Note, the crosscorrelation image of the point scatterer is smeared over a larger depth range than that of the Kirchhoff image. This is because the crosscorrelation of one trace with another smears the source wavelet into a longer wavelet, and also because the crosscorrelation migration kernel has poor resolution in the depth direction. Nevertheless, the crosscorrelation point-scatterer image is acceptable.

In practice, the trace at the master trace location and it’s two nearest neighbors were muted because the direct wave migration kernel in equation 15 has zero or nearly zero phase when \( A \approx B \). This is undesirable because any energy from these traces will be smeared uniformly throughout the model, not just at the buried source points. Also, a second derivative in time was applied to the crosscorrelogram traces to partly compensate for the smoothing effects of crosscorrelation and migration (Schuster et al., 2002).

Figure 13 is the same as Figure 12 except the source wavelet is a long random-time series. The crosscorrelation of traces collapses the ringy time series to an impulse-like wavelet so that the associated migration image in Figure 13c has good spatial resolution compared to the Kirchhoff image in Figure 13b.

In the previous examples, the scatterer exploded at time zero. Now, there are ten scatterers and all are assumed to explode at random times with a random time series as a source wavelet. The resulting data for 1 second is shown in Figure 14a. Figure 14b shows these data after crosscorrelation migration of 1 second of data, and roughly locates the location of the 10 point exploders. Repeating this crosscorrelation migration for fifteen data sets, each with 1 second of data generated from ten point scatterers with distinct random time histories, yields the stacked images in Figures 14c and 15. As expected, averaging the migration images tends to cancel migration noise and reinforce the energy at the location of the point exploders.

Finally, the fault-like structure denoted by stars in Figure 16 is assumed to emanate seismic energy randomly in time with random strength. This might approximate the situation where fluid is injected along a reservoir bed and seismic instru-
Figure 12: (Top) Synthetic 30 Hz data generated by an impulsive-like point exploder (*) at a depth of 1050 m. The point exploded at time zero. (Middle) Kirchhoff migration image, and (bottom) crosscorrelation migration image. The Kirchhoff image is better resolved partly because temporal crosscorrelation of traces will broaden the wavelet.
Figure 13: Same as previous figure, except a long random time series is used for the source wavelet that is excited at time zero. Note, that the crosscorrelation of traces collapses the ringy source wavelet into an impulsive-like wavelet, leading to a better resolved migration image in the crosscorrelogram image.
ments are passively monitoring the location of the injection front. Figure 16 shows the results after crosscorrelation migration of (middle) 1 second of data and (bottom) 40 stacks of 1 second records. The fault boundaries are much better delineated in the 40-stack migration image, although the resolution is much worse than that of an ordinary seismic survey. Figure 17 shows detailed contour images of these migration images.

Poor resolution of the crosscorrelation images is consistent with the poor resolution predicted by the crosscorrelation migration impulse response shown in Figure 18. A possibility for improving resolution is to measure the incidence angle of energy in the crosscorrelograms and use this angle as a constraint in smearing data into the model. This strategy is similar to that of ray-map migration or wavepath migration (Sun and Schuster, 2001), but it remains to be seen if this is a practical strategy with crosscorrelograms.

SUMMARY

A general methodology is presented for using correlated data to image source locations or reflector boundaries for \( v(x, y, z) \) media. Traces are crosscorrelated in time and weighted by the appropriate migration kernel, and summation over all geophone positions is carried out to give the migrated image (e.g., equation 7). Our analysis supports Claerbout’s conjecture: crosscorrelating a trace at \( A \) with one at \( B \) yields a trace with the ghost-direct correlation kinematically equivalent to a primary reflection generated by a source at \( A \) and recorded at \( B \). Both synthetic and field data examples are presented which highlight both the efficacy and weaknesses of II.

A key merit of interferometric imaging is the potential to image the reflectivity distribution and source locations from passive seismic data when the source location and wavelets are not known. For multiple sources with overlapping time histories, the source wavelets must be uncorrelated for successful II. In the case of autocorrelation migration, defocusing of the image from static errors and migration velocity errors can be significantly reduced by II.

The main disadvantage of II is the presence of virtual multiples in the correlated data, which can lead to severe migration artifacts. Therefore, coherent noise reduction should be applied to the correlated data prior to migration. Simultaneous use of the primary reflection and ghost reflection imaging conditions should also be used, as well as the blended imaging concept described in the text. Sometimes raw data can be time shifted to that of a direct or reflected arrival and then migrated according to an interferometric imaging condition (Sheley and Schuster, 2000). This avoids the need to correlate data altogether.
Figure 14: Similar to previous figure, except the source wavelets of ten point exploders (*) are generated by a random number generator. The middle figure shows the crosscorrelogram image computed from 1 second of data, while the bottom images shows the result after 15 stacks of 1-second data. The stacked image is better resolved because stacking tends to cancel noise and reinforce migration energy at the point exploder locations.
Figure 15: Similar to previous figure except the migration images are contoured.

It is interesting to note the analogy of inverting correlograms with that of constructing holograms. Light intensity images are used to reconstruct holograms, which are images of the object polluted by noisy interference terms. Seismic correlograms can also be used to reconstruct images of the object, but the other terms in equation 3 suggest that the data are polluted by noisy interference terms. However, we can design migration operators to reconstruct the object function (e.g., reflectivity) or information from the interference terms (e.g., source locations). As in holography, the challenge with seismic crosscorrelograms is to reduce the noise from the interference terms and enhance the signal from the events tuned to the migration operator.

References


Figure 16: (Top). Synthetic 30 Hz data generated by 55 point exploders located along a fault-like boundary. The points exploded at random times with random weighting amplitudes. (middle) Crosscorrelation migration image obtained from 1 second of data, and (bottom) crosscorrelation migration image after 40 stacks of 1-second data. The stacked image appears to be less noisy and a better approximation to the fault geometry delineated by stars.
Figure 17: Similar to previous figure except the migration images are contoured, and the fault boundary is delineated by a dashed line.
Figure 18: Isotime contours (in seconds) of the impulse response of the (top) cross-correlation migration and (bottom) prestack Kirchhoff migration operators. The + and * symbols represent the locations of the source and receiver, respectively, where the source location for the crosscorrelograms is the same as the master trace. The crosscorrelation migration operator is dominated by nearly vertical contours, so its resolution should be poorest in the vertical direction.


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APPENDIX: Stationary Phase Approximation

To mathematically justify, in a stationary-phase sense, the migration of free-surface reflections, we focus attention on the correlation $\tilde{\Phi}(A, B)_{\text{direct}_A - \text{ghost}_B} = -Re^{i\omega(\tau_{sA} - \tau_{sA'} + \tau_{A'} x + \tau_{Bx})}$ in equation 5, except we do not assume that the dominant contribution is a specular reflection at $A$. In this case the $\tilde{\Phi}(A, B)_{\text{direct}_A - \text{ghost}_B}$ correlation is given by an integral over the free-surface

$$
\tilde{\Phi}(A, B)_{\text{direct}_A - \text{ghost}_B} = -Re^{-i\omega\tau_{sA}} \int_{-\infty}^{\infty} e^{i\omega(\tau_{sA'} + \tau_{A'} x + \tau_{Bx})} dA',
$$

(16)

where geometrical spreading is ignored and $x$ is the location of the scatterer. Applying the stationary-phase approximation at high frequencies to the above integral yields

$$
\tilde{\Phi}(A, B)_{\text{direct}_A - \text{ghost}_B} \sim -CR e^{-i\omega\tau_{sA}} e^{i\omega(\tau_{sA_{\text{spec}}} + \tau_{A_{\text{spec}}x} + \tau_{Bx})},
$$

(17)

where $C$ is an asymptotic coefficient term (Bleistein, 1984) and the location $A_{\text{spec}}$ is the stationary value that satisfies the following equation:

$$
\nabla \tau_{sA} = -\nabla \tau_{Ax}.
$$

(18)

Here the derivatives are with respect to the $A$ variable. This equation is satisfied when $A = A_{\text{spec}}$ is the specular reflection point on the free surface as shown by the raypath $sA$ in Figure 2a; i.e., the upcoming ray angle from the source is equal and opposite to the reflection ray at the specular reflection point $A_{\text{spec}}$ on the free surface.

Applying the ghost migration kernel $e^{-i\omega(\tau_{A_{x'} + \tau_{Bx'}})}$ in equation 6 to $\tilde{\Phi}(A, B)_{\text{direct}_A - \text{ghost}_B}$ in equation 17 and integrating over all $A$ yields the migration image

$$
m(x') = -CR \int e^{i\omega(-\tau_{sA} + \tau_{sA_{\text{spec}}} + \tau_{A_{\text{spec}}x} + \tau_{Bx})} e^{-i\omega(\tau_{A_{x'}} + \tau_{Bx'})} dA,
$$

$$
= -CC'R e^{i\omega(-\tau_{sA} + \tau_{sA_{\text{spec}}} + \tau_{A_{\text{spec}}x} + \tau_{Bx} - \tau_{A_{x'}} - \tau_{B_{x'}})},
$$

(19)
where $A^*$ is the new stationary phase point and $C'$ is its associated asymptotic coefficient term. In this case, the stationary phase condition is

$$\nabla \tau_{sA} = -\nabla \tau_{Ax'},$$

(20)

which is the same as the previous one when the trial image point $x'$ is equal to the scatterer location $x$, so that $A^* = A_{spec}$. Thus, the exponent in equation 19 is zero. When the migrated image is summed over all locations of $B$ then this leads to constructive interference of migrated free-surface reflections at the scatterer location. Conversely, if the image point $x'$ is not coincident with $x$ then there will be mostly destructive superposition of migrated free-surface reflections away from the actual scatterer location.

The above analysis suggests that we can roughly think of the correlation kernel $\Phi_{gg'}$ as a trace generated by a source at $g$ and recorded at $g'$. In this case, the direct waves correlated with free-surface reflections play the role of primary reflections in the crosscorrelograms. The migration image of the point scatterer is obtained by a weighted spatial correlation of the crosscorrelograms, where the weights are the migration operators for a source at $g$ and a receiver at $g'$. This is the correlation equivalent of prestack migration, while weighting the correlation kernel $\bar{\Phi}_{gg}$ and summing over the $g$ index is the correlation equivalent of poststack migration.

Note that the migration operator does not depend on the source position or the scatterer location or depth so this procedure also applies to data generated by a random distribution of sources and a medium with many scatterers. It is straightforward to append a summation over scatterers to generalize this procedure to arbitrary reflector boundaries. A similar stationary phase analysis can be applied to the migration operator associated with the problem of determining the source location.