Rapid Calculation of the 3-D Recording Footprint

Zhaojun Liu

September 28, 1997

ABSTRACT

The Seismic Array Theorem is used to rapidly calculate the "recording footprint" noise for different source-receiver surveys. This theorem can be used for trial and error design of an optimal orthogonal shooting geometry. Here I present formulae for calculating the monochromatic recording footprint for marine surveys, in which the receivers are always shifted for each source position shift. I also calculate recording footprints for different seismic surveys devised by ARCO. Images of these recording footprints clearly distinguish the "good" surveys from the "bad" surveys.

INTRODUCTION

The "recording footprint" noise is the numerical noise introduced into the migration
section by the discrete sampling of wavefields measured over a finite-width aperture. By using the Seismic Array Theorem, Schuster (1997) presented formulae for the rapid calculation of the recording footprint for a land survey. The goal is to use this theorem to design a cost-effective seismic survey that optimizes the clarity of the migrated images.

Here I present formulae for calculating the monochromatic recording footprint of the marine survey, in which the receivers are always shifted with the source positions. I also present the calculated recording footprints for different seismic surveys used by ARCO. Images of these recording footprints clearly distinguish the ”good” surveys from the ”bad” surveys.

**Seismic Array Theorem**

The Array Theorem (Reynolds et al., 1989) is used by optical engineers to rapidly compute the optical impulse response of an aperture composed of a regular array of openings. This theorem has been modified for seismic migration (Schuster, 1997).

**Seismic Array Theorem : 1** Assume a 3-D seismic experiment where each shot shoots into a fixed array of geophone stations with area $L_x \times L_y$, below which is a point scatterer embedded in a homogeneous medium (see Figure 1). Assume that the source and the receiver distributions are separable, i.e., the 4-D sampling function $h'(\mathbf{x}_s, \mathbf{x}_g)$ can be decomposed into a concatenation of 2 2-D sampling functions $h(\mathbf{x}_s, \mathbf{x}_g) = h(\mathbf{x}_s)h(\mathbf{x}_g)$. Let $n(\mathbf{x})$ and $e(\mathbf{x})$ repre-
sent the north-south and east-west 1-D sampling combs, so that the 2-D sampling functions can be described by \( h(\mathbf{r}_s) = \int n(\mathbf{r}_s - \mathbf{r}')e(\mathbf{r}')d\mathbf{r}' \) and \( h(\mathbf{r}_g) = \int n(\mathbf{r}_g - \mathbf{r}')e(\mathbf{r}')d\mathbf{r}' \). Under the far-field approximation, the migrated image of a point scatterer is given by:

\[
    m(\mathbf{r}, \omega) = A \int \int h'(\mathbf{r}_s, \mathbf{r}_g) e^{-i(\mathbf{k}_s \cdot \mathbf{r}_s + \mathbf{k}_g \cdot \mathbf{r}_g)} d\mathbf{r}_s d\mathbf{r}_g,
    \\
    = A \int h(\mathbf{r}_s)e^{-i\mathbf{k}_s \cdot \mathbf{r}_s} d\mathbf{r}_s \int h(\mathbf{r}_g)e^{-i\mathbf{k}_g \cdot \mathbf{r}_g} d\mathbf{r}_g,
    \\
    \approx \tilde{N}(\mathbf{k}_s) \tilde{E}(\mathbf{k}_s) \tilde{N}(\mathbf{k}_g) \tilde{E}(\mathbf{k}_g),
\]

where \( \mathbf{k}_s = \mathbf{k}_g = \frac{k(\mathbf{r} - \mathbf{r}_0)}{r} \), \( k \) is the wavenumber, \( \mathbf{r} \) points to the image point, \( \mathbf{r}_0 \) points to the point scatterer, and \( r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + z_0^2} \). For a 1-D sampling comb, \( \tilde{N}(\mathbf{k}_g) = \frac{\sin(k_{gy}L_y/2)}{\sin(k_{gy}dy_y/2)} \), \( \tilde{E}(\mathbf{k}_g) = \frac{\sin(k_{gx}L_x/2)}{\sin(k_{gx}dx_x/2)} \), \( \tilde{N}(\mathbf{k}_s) = \frac{\sin(k_{sy}L_y/2)}{\sin(k_{sy}dy_y/2)} \), and \( \tilde{E}(\mathbf{k}_s) = \frac{\sin(k_{sx}L_x/2)}{\sin(k_{sx}dx_x/2)} \). Here \( k_{gx} = k_{sx} = k(x-x_0)/r, k_{gy} = k_{sy} = k(y-y_0)/r, dx_x \) and \( dy_y \) are the source interval in \( x \) and \( y \) direction, and \( dx_g \) and \( dy_g \) are the receiver interval in \( x \) and \( y \) direction.

For a marine seismic survey, the receiver locations are source-dependent so that the recording cables are always shifted with any shift of the source group. Therefore, the 4-D sampling function can be decomposed as \( h'(\mathbf{r}_s, \mathbf{r}_g) = h(\mathbf{r}_s)h(\mathbf{r}_g - \mathbf{r}_s) \).

**Seismic Array Theorem**: Assume a 3-D seismic experiment where each shot shoots
into an array of geophone stations, below which is a point scatterer embedded in a homogeneous medium. Assume the 4-D sampling function $h'(r_s, r_g)$ can be decomposed into a concatenation of 2 2-D sampling functions $h'(r_s, r_g) = h(r_s)h(r_g - r_s)$, which accounts for a shift in the geophone array w/r to shift the in the source location. Under the far-field approximation, the migrated image of a point scatterer is given by:

\[
m(x, \omega) = A \int \int h'(r_s, r_g) e^{-i (\kappa_s \cdot r_s + \kappa_g \cdot r_g)} dr_s dr_g,
\]

\[
= A \int h(r_s) e^{-i \kappa_s \cdot r_s} dr_s \int h(r_g - r_s) e^{-i \kappa_g \cdot r_g} dr_g,
\]

\[
= A \int h(r_s) e^{-i \kappa_s \cdot r_s} dr_s \int h'(r_g) e^{-i (\kappa_g + \kappa_s) \cdot r_g} dr_g,
\]

\[
= A \int h(r_s) e^{-i \kappa_s \cdot r_s} dr_s \int h'(r_g) e^{-i \kappa_g \cdot r_g} dr_g,
\]

\[
= A \int h(r_s) e^{-i \kappa_s \cdot r_s} dr_s \int h'(r_g) e^{-i \kappa_g \cdot r_g} dr_g.
\]

\[
\approx \bar{N}(2\kappa_s) \bar{E}(2\kappa_s) \bar{N}(\kappa_g) \bar{E}(\kappa_g).
\]

(2)

With the use of this Seismic Array Theorem, the point scatterer migration response of a regular recording geometry can be computed by a multiplication of four analytical expressions.
Numerical Examples

Example 1: Land Seismic Survey

Figures 2 and 3 show the source and receiver geometries, respectively, for three different seismic surveys. In these land surveys, all receivers are activated for each shot. The recording area of each survey is 7 kft x 7 kft, and the geophone and source sampling intervals are given by \((dx_g, dy_g)\) and \((dx_s, dy_s)\), respectively. In survey I, \(dx_g = dy_s = 0.24\) kft, \(dy_g = dx_s = 0.04\) kft; and in survey II, \(dx_g = dy_s = 0.12\) kft, \(dy_g = dx_s = 0.04\) kft; and in survey III, \(dx_g = dy_s = 0.12\) kft, \(dy_g = dx_s = 0.02\) kft. So, about 28,000,000 traces are computed for each of the three surveys. It is assumed that the wavelength of the source is 150 ft, and the point scatterer is located at \((0, 0, z_0 = 4\) kft).

Figure 4 shows the monochromatic migrated images of a buried point scatterer; i.e., the "recording footprint" of the surveys calculated by the Seismic Array Theorem. Figure 5 shows the 3-D picture of the recording footprint more clearly, where the recording footprint for survey II has weaker artifacts than the others. Artifacts are defined as migration energy located outside the main pulse centered at \((0, 0, 4.0\) kft). For this frequency, survey II should produce the least recording footprint noise than the other surveys.

Table 1. Recording parameters for three land surveys.
<table>
<thead>
<tr>
<th>Survey</th>
<th>$dx_s$</th>
<th>$dy_s$</th>
<th>$dx_g$</th>
<th>$dy_g$</th>
<th># of traces</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.04 kft</td>
<td>0.24 kft</td>
<td>0.24 kft</td>
<td>0.04 kft</td>
<td>27,878,400</td>
</tr>
<tr>
<td>II</td>
<td>0.04 kft</td>
<td>0.12 kft</td>
<td>0.12 kft</td>
<td>0.04 kft</td>
<td>29,160,000</td>
</tr>
<tr>
<td>III</td>
<td>0.02 kft</td>
<td>0.12 kft</td>
<td>0.12 kft</td>
<td>0.02 kft</td>
<td>29,160,000</td>
</tr>
</tbody>
</table>

**Example 2: ARCO’s Marine Seismic Survey**

Figure 6 shows four different marine surveys with o representing the source location and + representing the receiver location. The darkened receiver symbols + indicate the geophones activated for a single shot. The darkened shot symbols o indicate the shot locations that shoot into the same receiver array indicated by darkened receiver symbols +. Figures 7 and 8 show the recording footprint images for the four seismic surveys, where it appears that surveys III and IV have the weakest sidelobe energy and, therefore, are better survey choices than the first two.

<table>
<thead>
<tr>
<th>Survey</th>
<th>$dx_s$</th>
<th>$dy_s$</th>
<th>$dx_g$</th>
<th>$dy_g$</th>
<th>Cable Length</th>
<th># of Cable Lines</th>
<th># of Traces</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>25m</td>
<td>25m</td>
<td>25m</td>
<td>3000m</td>
<td>1</td>
<td>1</td>
<td>1,600,000</td>
</tr>
<tr>
<td>II</td>
<td>75m</td>
<td>300</td>
<td>25m</td>
<td>100m</td>
<td>6</td>
<td>6</td>
<td>1,600,000</td>
</tr>
<tr>
<td>III</td>
<td>200m</td>
<td>750m</td>
<td>50m</td>
<td>750m</td>
<td>8</td>
<td>8</td>
<td>1,600,000</td>
</tr>
<tr>
<td>IV</td>
<td>250m</td>
<td>250m</td>
<td>50m</td>
<td>1500mX2000m</td>
<td>1</td>
<td>1</td>
<td>1,600,000</td>
</tr>
</tbody>
</table>
CONCLUSIONS

The Seismic Array Theorem can be used to rapidly compute the recording footprint of a 3-D seismic survey. I present the formulae which represents the monochromatic migrated image of a point scatterer, which is a simple analytical expression. This formula can be used in a trial and error method to choose the optimal source-receiver intervals that minimize the recording footprint noise away from the point scatterer. As examples, I calculated the point scatterer migrated images for different ARCO recording geometries. Images of these recording footprints clearly distinguish the "good" footprints from the "bad" footprints. My next step is to seek an optimization algorithm that automatically chooses the optimal recording geometry that minimize the footprint.

ACKNOWLEDGEMENTS

I would like to thank Steve Moore of ARCO Inc. for providing the information about the 3-D marine survey.

REFERENCES

p.59-71.

Figure 1: Point scatterer at $r_0(x_0, y_0, z_0)$ and image point at $(x, y, z_0)$. 
Figure 2: The source geometry for the three different land surveys.
Figure 3: The receiver geometry for the three different land surveys.
Figure 4: Recording footprint magnitude of the three surveys evaluated at depth of 4 kft.
Figure 5: Mesh images of the recording footprint magnitudes for the three surveys. The survey II image has the fewest migration artifacts.
Figure 6: The source-receiver geometries for the four marine seismic surveys. The darkened receiver "+" symbols indicate the geophones activated for a single shot indicated by a darkened "o" symbol. The darkened shot "o" symbols indicate the shot locations that shoot into the same receiver array indicated by darkened "+" symbols. These receiver arrays are shifted by the same amount as a shift in the shot arrays.
Figure 7: The recording footprints (i.e., the point scatterer migration response evaluated at the scatterer depth) for the four marine surveys in Figure 6.
Figure 8: Mesh images of the recording footprints shown in the previous figure. The ideal migration image is a single pulse at the center of the above grid.